## Advanced Financial Management

Friday 7 September 2018


Time allowed: 3 hours 15 minutes

This question paper is divided into two sections:
Section A - This ONE question is compulsory and MUST be attempted
Section B - BOTH questions are compulsory and MUST be attempted
Formulae and tables are on pages 10-14.
Do NOT open this question paper until instructed by the supervisor.

This question paper must not be removed from the examination hall.


## Think Ahead

The Association of Chartered Certified

Accountants

This is a blank page.
The question paper begins on page 3.

## Section A - This ONE question is compulsory and MUST be attempted

1 Washi Co is a large, unlisted company based in Japan and its local currency is the Japanese Yen (JPY). It manufactures industrial equipment and parts. Initially Washi Co's customers consisted of other Japanese companies, but over the last 12 years it has expanded into overseas markets and also sources its materials from around the world. The company's board of directors (BoD) believes that the strategy of overseas investments, through subsidiary companies, branches and joint ventures, has directly led to the company's substantial increase in value in the past few years.

Washi Co's BoD is considering investing in a project based in Airone, whose currency is the Airone Rand (ARD). It believes that the project will be an important addition to the company's portfolio of investments, because Washi Co does not currently have a significant presence in the part of the world where Airone is located. It is intended that the project will commence in one year's time. Details of the project are given below.

Washi Co intends to finance the project through proceeds from an agreed sale of a small European subsidiary, with any remaining funding requirement being met by additional debt finance issued in Japanese Yen. The company is due to receive the proceeds from the sale of a European subsidiary company in six months' time and it will then invest these funds in short-dated Japanese treasury bills for a further six months before they are needed for the project. Washi Co has a centralised treasury department, which hedges expected future cash flows against currency fluctuations.

## Funding and financial information

The agreed proceeds from the sale of the European subsidiary company receivable in six months' time are Euro (EUR) 80 million. The BoD is concerned about a negative fluctuation in EUR/JPY rate between now and in six months when the EUR 80 million will be received. Therefore, it has asked Washi Co's treasury department to hedge the expected receipt using one of currency forwards, currency futures or exchange traded currency options. Washi Co's treasury department has obtained the following information:

$$
\text { JPY per EUR } 1 \quad \text { ARD per EUR } 1
$$

Spot
129.2-132.4
92.7-95.6

Six-month forward rate
125-3-128.6
Currency futures (contract size EUR 125,000, quotation JPY per EUR 1)
Four-month expiry 126.9
Seven-month expiry $125 \cdot 2$

Currency options (contract size EUR 125,000, exercise price quotation: JPY per EUR 1, premium quotation: JPY per EUR 1)

At an exercise price of JPY 126.0 per EUR 1

|  | Four-month expiry | Seven-month expiry |
| :--- | :---: | :---: |
| Calls | 2.3 | 2.6 |
| Puts | 3.4 | 3.8 |

Annualised yield on short-dated Japanese treasury bills 1•20\%
Airone's annual inflation rate is $9 \%$ currently, but has fluctuated markedly in the last five years. The Japanese annual inflation rate is $1.5 \%$ and has been stable for many years.
Pato Bank has offered Washi Co the possibility of using over-the-counter options to hedge the EUR receipt instead of exchange traded currency options.

## Airone project information

A member of Washi Co's finance team has produced the following estimates of the Airone project which is expected to last for four years. The estimates are based on the notes given below but not on the further information. The estimates have been checked and verified independently for their numerical accuracy.

All figures are in ARD millions

| Project year | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sales revenue |  | 13,000 | 30,800 | 32,300 | 4,500 |
| Costs |  | $(10,200)$ | $(24,200)$ | $(24,500)$ | $(3,200)$ |
| Tax allowable depreciation |  | $(1,000)$ | $(1,000)$ | $(1,000)$ | $(1,000)$ |
| Pre-tax profits |  | 1,800 | 5,600 | 6,800 | 300 |
| Tax at 15\% |  | (270) | (840) | $(1,020)$ | (45) |
| Tax allowable depreciation |  | 1,000 | 1,000 | 1,000 | 1,000 |
| Working capital | (400) |  |  |  | 400 |
| Investment in buildings | $(5,750)$ |  |  |  |  |
| Investment in machinery | $(4,000)$ |  |  |  |  |
| Cash flows in ARD | $(10,150)$ | 2,530 | 5,760 | 6,780 | 1,655 |

## Notes (incorporated into the estimates above):

1. The estimates are based on using the end of the first year, when the project commences, as the start of the project (year 0). The numbers are given in ARD million (m).
2. The total investment required for the project is ARD 10,150m and separated into buildings, machinery and working capital in the table above. The machinery is eligible for tax allowable depreciation on a straight-line basis and the working capital is redeemable at the end of the project.
3. The impact of inflation has been incorporated into the sales revenue and cost figures, at Airone's current annual inflation figures.
4. Corporation tax has been included based on Airone's annual rate of $15 \%$. The tax is payable in the year that the tax liability arises.

## Further information (not incorporated into the estimates above):

1. Undertaking the Airone project will result in lost sales for Washi Co. These sales would have generated a pre-tax contribution of JPY 110 m in the first year of the project, rising by the Japanese rate of inflation in the following years 2 to 4 of the project.
2. The Airone project costs include components which are made in Japan by Washi Co and would be imported to the Airone project. The pre-inflation revenues generated from the sale of the components are estimated to be as follows:

In JPY millions

| Project year | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Components revenue | 1,200 | 2,400 | 2,500 | 300 |

These revenues are expected to increase by the Japanese inflation rate in years 2 to 4 of the project. The contribution which Washi Co expects to earn on these components is $25 \%$ of revenue.
3. The Japanese annual corporation tax rate is $30 \%$ and tax is payable in the year that the tax liability arises. A bilateral tax treaty exists between Japan and Airone, which permits offset of overseas tax against any Japanese tax liability on overseas earnings.
4. Washi Co's finance department has estimated a cost of capital of $12 \%$ to be used as a discount rate for the project.

## Required:

(a) Discuss how investing in overseas projects may enable Washi Co to gain competitive advantage over its competitors, who only invest in domestic projects.
(b) Discuss the advantages and drawbacks of exchange traded option contracts compared with over-the-counter options.
(c) Prepare a report for the board of directors of Washi Co which:
(i) Estimates the expected amount of JPY receivable under each hedge choice and the additional debt finance needed to fund the Airone project for the preferred hedge choice;
(ii) Estimates the net present value of the Airone project in Japanese Yen, based on the end of year one being the start of the project (year 0 );
(9 marks)
(iii) Evaluates the preferred hedge choice made, the debt finance needed and whether the Airone project should be undertaken, considering both financial and non-financial factors.
(8 marks)
Professional marks will be awarded in part (c) for the format, structure and presentation of the report.
(d) Washi Co's chief operations officer (COO) has suggested that it would be more beneficial for the company to let its major subsidiary companies have their own individual treasury departments, instead of having one centralised treasury department for the whole company.

Required:
Discuss the validity of the COO's suggestion.

## Section B - BOTH questions are compulsory and MUST be attempted

## 2 Tillinton Co

Tillinton Co is a listed company which has traditionally manufactured children's clothing and toys with long lives. Five years ago, it began manufacturing electronic toys and has since made significant investment in development and production facilities. The first electronic toys which Tillinton Co introduced into the market were received very well, partly as it was seen to be ahead of its competitors in making the most of the technology available.

The country where Tillinton Co is listed has seen a significant general increase in share prices over the last three years, with companies in the electronic goods sector showing particularly rapid increases.

## Statement by Tillinton Co's chief executive

Assume it is now September 20X3. Tillinton Co's annual report for the year ended 31 March 20X3 has just been published. Its chief executive commented when announcing the company's results:
'I am very pleased to report that revenue and gross profits have shown bigger increases than in 20X2, resulting in higher post-tax earnings and our company being able to maintain increases in dividends. The sustained increase in our share price clearly demonstrates how happy investors are with us. Our cutting-edge electronic toys continue to perform well and justify our sustained investment in them. Our results have also benefited from improvements in operational efficiencies for our older ranges and better working capital management. We are considering the development of further ranges of electronic toys for children, or developing other electronic products for adults. If necessary, we may consider scaling down or selling off our operations for some of our older products.'

Steph Slindon represents an institutional investor who holds shares in Tillinton Co. Steph is doubtful whether its share price will continue to increase, because she thinks that Tillinton Co's situation may not be as good as its chief executive suggests and because she believes that current share price levels generally may not be sustainable.

## Financial information

Extracts from Tillinton Co's financial statements for the last three years and other information about it are given below.
Tillinton Co statement of profit or loss in years ending 31 March (all amounts in \$m)

|  | 20X1 | 20X2 | 20x3 |
| :---: | :---: | :---: | :---: |
| Sales revenue | 1,385 | 1,636 | 1,914 |
| Gross profit | 381 | 451 | 528 |
| Operating profit | 205 | 252 | 300 |
| Finance costs | (46) | (50) | (66) |
| Profit before tax | 159 | 202 | 234 |
| Taxation | (40) | (51) | (65) |
| Profit after tax | 119 | 151 | 169 |
| Dividends | (60) | (72) | (84) |

Tillinton Co statement of financial position in years ending 31 March
(all amounts in \$m)

|  | 20X1 | 20X2 | 20X3 |
| :---: | :---: | :---: | :---: |
| Non-current assets | 2,070 | 2,235 | 2,449 |
| Cash and cash equivalents | 10 | 15 | 15 |
| Other current assets | 150 | 130 | 125 |
| Total non-current and current assets | 2,230 | 2,380 | 2,589 |
| Equity |  |  |  |
| Ordinary shares (\$0.50) | 400 | 400 | 400 |
| Reserves | 805 | 884 | 969 |
| Total equity | 1,205 | 1,284 | 1,369 |
| Non-current liabilities | 920 | 970 | 1,000 |
| Current liabilities | 105 | 126 | 220 |
| Total equity and liabilities | 2,230 | 2,380 | 2,589 |
| Other information |  |  |  |
| Market price per \$0.50 share (in \$, \$2.50 at |  |  |  |
| 31 March 20X0, \$5•06 in September 20X3) | $2 \cdot 76$ | 3.49 | $4 \cdot 44$ |
| Earnings per share (\$) | $0 \cdot 15$ | $0 \cdot 19$ | 0.21 |
| Dividend per share (\$) | 0.075 | 0.09 | $0 \cdot 105$ |
| Analysis of revenue |  |  |  |
| Electronic toys | 249 | 319 | 390 |
| Non-electronic toys | 302 | 350 | 404 |
| Clothing | 834 | 967 | 1,120 |
|  | 1,385 | 1,636 | 1,914 |
| Analysis of gross profit |  |  |  |
| Electronic toys | 100 | 112 | 113 |
| Non-electronic toys | 72 | 88 | 105 |
| Clothing | 209 | 251 | 310 |
|  | 381 | 451 | 528 |

Note: None of Tillinton Co's loan finance in 20X3 is repayable within one year.

## Required:

(a) Evaluate Tillinton Co's performance and business prospects in the light of the chief executive's comments and Steph Slindon's concerns. Provide relevant calculations for ratios and trends to support your evaluation.

Note: 10 marks are available for the calculations.
(20 marks)
(b) Discuss how behavioural factors may have resulted in Tillinton Co's share price being higher than is warranted by a rational analysis of its position.

## 3 Selorne Co

Selorne Co is one of the biggest removal companies in Pauland, offering home and business removals. It has a number of long-term contracts with large businesses, although it has not won any new major contracts in the last two years. Selorne Co is listed on Pauland's stock market for smaller companies. Selorne Co is financed by a mixture of equity and short and long-term debt, but its gearing level is below the average for its sector.

Selorne Co has four executive directors, who each own 20\% of the company's share capital, with the other 20\% owned by external shareholders. Selorne Co has paid a constant dividend since it has been listed and its share price has risen slightly over the last three years.

Selorne Co is based in a number of the large cities and towns in Pauland and owns the majority of the sites where it is located. Many of its employees have worked for the company for a long time. Drivers of the lorries used by Selorne Co are required to have a special, heavy vehicles licence. Salary levels at Selorne Co are relatively high compared with other companies in the sector.

## Chawon Co

Selorne Co is currently considering making a bid for Chawon Co, an unlisted company specialising in distribution and delivery services. Chawon Co is owned $100 \%$ by its founder, Chris Chawon. Chawon Co has built up a portfolio of small contracts over time. It has made unsuccessful bids for two larger contracts over the last 12 months, the bids being rejected primarily because Chawon Co was not felt to be big enough to be able to guarantee the level of service required.

Chawon Co is based in many of the same cities and towns where Selorne Co is located, although Chawon's premises are all rented. The drivers of Chawon's vehicles do not require a heavy vehicles licence. Chawon Co has a few long-serving employees who are mostly centre managers. Most of its drivers and staff, however, stay at Chawon Co for only a short time. Salary levels are low, although Chawon Co pays high levels of overtime and high bonuses if target profit levels are achieved. Chawon Co is highly geared, leading to recent media speculation about its financial viability.

## Terms of bid for Chawon Co

In initial discussions about the acquisition, Chris Chawon indicated that he would prefer the consideration to be a share-for-share exchange, the terms being one Chawon Co share for five Selorne Co shares.

Chawon Co has 2 million $\$ 1$ shares in issue, and Selorne Co has 50 million $\$ 0.50$ shares in issue. Each Selorne Co share is currently trading at $\$ 6 \cdot 50$, which is a multiple of 8 of its free cash flow to equity. The multiple of 8 can be assumed to remain unchanged if the acquisition takes place. Chawon Co's free cash flow to equity is currently estimated at $\$ 7$ million, with an expected annual growth rate of $3 \%$, and it is expected to generate a return on equity of $15 \%$.

Chris Chawon expects that the total free cash flows to equity of the combined company will increase by $\$ 5$ million due to synergy benefits. He believes that Selorne Co will be able to win more contracts because it is larger and because it will be diversifying the services which it offers. He also believes that significant operational synergies can be achieved, pointing out the time Selorne Co drivers spend idle during the winter months when removal activity is traditionally lower. Chris Chawon believes that he can achieve the synergies if he is given management responsibility for the operational reorganisation, including dealing with the staff employment and retention issues. Chris Chawon thinks that synergies could also be achieved in central administration and in premises costs.
The chief executive and the finance director of Selorne Co are in favour of bidding for Chawon Co. However, one of the other executive directors is opposed to the bid. He is sceptical about the level of synergies which can be achieved and does not want Chris Chawon to be brought into the management of Selorne Co. He suggests that if the bid is to go ahead, it should be a cash offer rather than a share exchange. Selorne Co's chief executive has responded that Chris Chawon is likely to ask for a higher equivalent price if the purchase is for cash.

## Financing the bid for Chawon Co

Selorne Co's finance director has pointed out that Selorne Co will need additional funding if Chawon Co is purchased for cash. He has suggested that there may be a number of possible sources of finance:

- A rights issue
- A fixed rate, long-term, bank loan
- A three-year, unsecured, mezzanine loan facility
- Convertible debt, with conversion rights being exercisable in five years' time


## Required:

(a) (i) Estimate the equity value of the combined company and the expected additional value arising from the combination of Selorne Co and Chawon Co.
(ii) Estimate the share of the gain from the combination created for Chris Chawon and the share of the gain created for Selorne Co's shareholders and comment on your results.
(b) Evaluate how reliable the estimates of the synergies for the combined company are likely to be and discuss the factors which may prevent the forecast synergies from being achieved.
(c) Discuss the factors which Selorne Co's board will consider when determining which source or sources of finance are chosen to finance a possible cash bid for the share capital of Chawon Co.

## Formulae

Modigliani and Miller Proposition 2 (with tax)

$$
k_{e}=k_{e}^{i}+(1-T)\left(k_{e}^{i}-k_{d}\right) \frac{V_{d}}{V_{e}}
$$

## The Capital Asset Pricing Model

$$
\mathrm{E}\left(\mathrm{r}_{\mathrm{i}}\right)=\mathrm{R}_{\mathrm{f}}+\beta_{\mathrm{i}}\left(\mathrm{E}\left(\mathrm{r}_{\mathrm{m}}\right)-\mathrm{R}_{\mathrm{f}}\right)
$$

The asset beta formula

$$
\beta_{\mathrm{a}}=\left[\frac{\mathrm{V}_{\mathrm{e}}}{\left(\mathrm{~V}_{\mathrm{e}}+\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})\right)} \beta_{\mathrm{e}}\right]+\left[\frac{\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})}{\left(\mathrm{V}_{\mathrm{e}}+\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})\right)} \beta_{\mathrm{d}}\right]
$$

The Growth Model

$$
P_{o}=\frac{D_{0}(1+g)}{\left(r_{e}-g\right)}
$$

## Gordon's growth approximation

$$
\mathrm{g}=\mathrm{br} \mathrm{r}_{\mathrm{e}}
$$

The weighted average cost of capital

$$
\text { WACC }=\left[\frac{V_{e}}{V_{e}+V_{d}}\right] k_{e}+\left[\frac{V_{d}}{V_{e}+V_{d}}\right] k_{d}(1-T)
$$

## The Fisher formula

$$
(1+i)=(1+r)(1+h)
$$

Purchasing power parity and interest rate parity

$$
S_{1}=S_{0} \times \frac{\left(1+h_{c}\right)}{\left(1+h_{b}\right)} \quad F_{0}=S_{0} \times \frac{\left(1+i_{c}\right)}{\left(1+i_{b}\right)}
$$

Modified Internal Rate of Return

$$
M I R R=\left[\frac{P V_{R}}{P V_{I}}\right]^{\frac{1}{n}}\left(1+r_{e}\right)-1
$$

The Black-Scholes option pricing model

$$
c=P_{a} N\left(d_{1}\right)-P_{e} N\left(d_{2}\right) e^{-r t}
$$

Where:

$$
\begin{aligned}
& d_{1}=\frac{\ln \left(P_{a} / P_{e}\right)+\left(r+0.5 s^{2}\right) t}{s \sqrt{t}} \\
& d_{2}=d_{1}-s \sqrt{t}
\end{aligned}
$$

The Put Call Parity relationship

$$
p=c-P_{a}+P_{e} e^{-r t}
$$

## Present Value Table

Present value of 1 i.e. $(1+r)^{-n}$
Where $r=$ discount rate
$\mathrm{n}=$ number of periods until payment
Discount rate (r)
Periods

| (n) | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ | $7 \%$ | $8 \%$ | $9 \%$ | $10 \%$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.990 | 0.980 | 0.971 | 0.962 | 0.952 | 0.943 | 0.935 | 0.926 | 0.917 | 0.909 | 1 |
| 2 | 0.980 | 0.961 | 0.943 | 0.925 | 0.907 | 0.890 | 0.873 | 0.857 | 0.842 | 0.826 | 2 |
| 3 | 0.971 | 0.942 | 0.915 | 0.889 | 0.864 | 0.840 | 0.816 | 0.794 | 0.772 | 0.751 | 3 |
| 4 | 0.961 | 0.924 | 0.888 | 0.855 | 0.823 | 0.792 | 0.763 | 0.735 | 0.708 | 0.683 | 4 |
| 5 | 0.951 | 0.906 | 0.863 | 0.822 | 0.784 | 0.747 | 0.713 | 0.681 | 0.650 | 0.621 | 5 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 0.942 | 0.888 | 0.837 | 0.790 | 0.746 | 0.705 | 0.666 | 0.630 | 0.596 | 0.564 | 6 |
| 7 | 0.933 | 0.871 | 0.813 | 0.760 | 0.711 | 0.665 | 0.623 | 0.583 | 0.547 | 0.513 | 7 |
| 8 | 0.923 | 0.853 | 0.789 | 0.731 | 0.677 | 0.627 | 0.582 | 0.540 | 0.502 | 0.467 | 8 |
| 9 | 0.914 | 0.837 | 0.766 | 0.703 | 0.645 | 0.592 | 0.544 | 0.500 | 0.460 | 0.424 | 9 |
| 10 | 0.905 | 0.820 | 0.744 | 0.676 | 0.614 | 0.558 | 0.508 | 0.463 | 0.422 | 0.386 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 0.896 | 0.804 | 0.722 | 0.650 | 0.585 | 0.527 | 0.475 | 0.429 | 0.388 | 0.350 | 11 |
| 12 | 0.887 | 0.788 | 0.701 | 0.625 | 0.557 | 0.497 | 0.444 | 0.397 | 0.356 | 0.319 | 12 |
| 13 | 0.879 | 0.773 | 0.681 | 0.601 | 0.530 | 0.469 | 0.415 | 0.368 | 0.326 | 0.290 | 13 |
| 14 | 0.870 | 0.758 | 0.661 | 0.577 | 0.505 | 0.442 | 0.388 | 0.340 | 0.299 | 0.263 | 14 |
| 15 | 0.861 | 0.743 | 0.642 | 0.555 | 0.481 | 0.417 | 0.362 | 0.315 | 0.275 | 0.239 | 15 |


| (n) | $11 \%$ | $12 \%$ | $13 \%$ | $14 \%$ | $15 \%$ | $16 \%$ | $17 \%$ | $18 \%$ | $19 \%$ | $20 \%$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.901 | 0.893 | 0.885 | 0.877 | 0.870 | 0.862 | 0.855 | 0.847 | 0.840 | 0.833 | 1 |
| 2 | 0.812 | 0.797 | 0.783 | 0.769 | 0.756 | 0.743 | 0.731 | 0.718 | 0.706 | 0.694 | 2 |
| 3 | 0.731 | 0.712 | 0.693 | 0.675 | 0.658 | 0.641 | 0.624 | 0.609 | 0.593 | 0.579 | 3 |
| 4 | 0.659 | 0.636 | 0.613 | 0.592 | 0.572 | 0.552 | 0.534 | 0.516 | 0.499 | 0.482 | 4 |
| 5 | 0.593 | 0.567 | 0.543 | 0.519 | 0.497 | 0.476 | 0.456 | 0.437 | 0.419 | 0.402 | 5 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 0.535 | 0.507 | 0.480 | 0.456 | 0.432 | 0.410 | 0.390 | 0.370 | 0.352 | 0.335 | 6 |
| 7 | 0.482 | 0.452 | 0.425 | 0.400 | 0.376 | 0.354 | 0.333 | 0.314 | 0.296 | 0.279 | 7 |
| 8 | 0.434 | 0.404 | 0.376 | 0.351 | 0.327 | 0.305 | 0.285 | 0.266 | 0.249 | 0.233 | 8 |
| 9 | 0.391 | 0.361 | 0.333 | 0.308 | 0.284 | 0.263 | 0.243 | 0.225 | 0.209 | 0.194 | 9 |
| 10 | 0.352 | 0.322 | 0.295 | 0.270 | 0.247 | 0.227 | 0.208 | 0.191 | 0.176 | 0.162 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 0.317 | 0.287 | 0.261 | 0.237 | 0.215 | 0.195 | 0.178 | 0.162 | 0.148 | 0.135 | 11 |
| 12 | 0.286 | 0.257 | 0.231 | 0.208 | 0.187 | 0.168 | 0.152 | 0.137 | 0.124 | 0.112 | 12 |
| 13 | 0.258 | 0.229 | 0.204 | 0.182 | 0.163 | 0.145 | 0.130 | 0.116 | 0.104 | 0.093 | 13 |
| 14 | 0.232 | 0.205 | 0.181 | 0.160 | 0.141 | 0.125 | 0.111 | 0.099 | 0.088 | 0.078 | 14 |
| 15 | 0.209 | 0.183 | 0.160 | 0.140 | 0.123 | 0.108 | 0.095 | 0.084 | 0.074 | 0.065 | 15 |

Present value of an annuity of 1 i.e. $\frac{1-(1+r)^{-n}}{r}$

| Where | $r=$ discount rate |
| :--- | :--- |
|  | $n=$ number of periods |

Discount rate (r)
Periods

| ( n ) | 1\% | 2\% | 3\% | 4\% | 5\% | 6\% | 7\% | 8\% | 9\% | 10\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.990 | 0.980 | 0.971 | 0.962 | 0.952 | 0.943 | 0.935 | 0.926 | 0.917 | 0.909 | 1 |
| 2 | 1.970 | 1.942 | 1.913 | 1.886 | 1.859 | 1.833 | 1.808 | 1.783 | 1.759 | 1.736 | 2 |
| 3 | $2 \cdot 941$ | $2 \cdot 884$ | $2 \cdot 829$ | $2 \cdot 775$ | $2 \cdot 723$ | $2 \cdot 673$ | $2 \cdot 624$ | $2 \cdot 577$ | 2.531 | $2 \cdot 487$ | 3 |
| 4 | 3.902 | 3.808 | 3.717 | 3.630 | 3.546 | 3.465 | $3 \cdot 387$ | $3 \cdot 312$ | 3.240 | $3 \cdot 170$ | 4 |
| 5 | 4.853 | $4 \cdot 713$ | 4.580 | 4.452 | $4 \cdot 329$ | $4 \cdot 212$ | $4 \cdot 100$ | 3.993 | 3.890 | $3 \cdot 791$ | 5 |
| 6 | $5 \cdot 795$ | $5 \cdot 601$ | $5 \cdot 417$ | $5 \cdot 242$ | 5.076 | $4 \cdot 917$ | $4 \cdot 767$ | $4 \cdot 623$ | $4 \cdot 486$ | 4.355 | 6 |
| 7 | 6.728 | 6.472 | 6.230 | 6.002 | 5.786 | $5 \cdot 582$ | $5 \cdot 389$ | $5 \cdot 206$ | 5.033 | $4 \cdot 868$ | 7 |
| 8 | 7.652 | 7.325 | 7.020 | 6.733 | 6.463 | $6 \cdot 210$ | 5.971 | $5 \cdot 747$ | 5.535 | $5 \cdot 335$ | 8 |
| 9 | 8.566 | $8 \cdot 162$ | $7 \cdot 786$ | 7.435 | $7 \cdot 108$ | $6 \cdot 802$ | $6 \cdot 515$ | $6 \cdot 247$ | 5.995 | 5.759 | 9 |
| 10 | $9 \cdot 471$ | 8.983 | 8.530 | $8 \cdot 111$ | $7 \cdot 722$ | $7 \cdot 360$ | $7 \cdot 024$ | $6 \cdot 710$ | $6 \cdot 418$ | $6 \cdot 145$ | 10 |
| 11 | $10 \cdot 368$ | 9.787 | $9 \cdot 253$ | 8.760 | $8 \cdot 306$ | 7.887 | $7 \cdot 499$ | $7 \cdot 139$ | 6.805 | 6.495 | 11 |
| 12 | $11 \cdot 255$ | $10 \cdot 575$ | 9.954 | $9 \cdot 385$ | $8 \cdot 863$ | 8.384 | 7.943 | 7.536 | $7 \cdot 161$ | 6.814 | 12 |
| 13 | $12 \cdot 134$ | $11 \cdot 348$ | $10 \cdot 635$ | 9.986 | 9.394 | 8.853 | 8.358 | 7.904 | $7 \cdot 487$ | $7 \cdot 103$ | 13 |
| 14 | 13.004 | $12 \cdot 106$ | 11.296 | $10 \cdot 563$ | $9 \cdot 899$ | 9.295 | $8 \cdot 745$ | 8.244 | 7.786 | $7 \cdot 367$ | 14 |
| 15 | $13 \cdot 865$ | $12 \cdot 849$ | 11.938 | $11 \cdot 118$ | $10 \cdot 380$ | $9 \cdot 712$ | $9 \cdot 108$ | 8.559 | 8.061 | $7 \cdot 606$ | 15 |
| ( n ) | 11\% | 12\% | 13\% | 14\% | 15\% | 16\% | 17\% | 18\% | 19\% | 20\% |  |
| 1 | 0.901 | 0.893 | 0.885 | 0.877 | 0.870 | $0 \cdot 862$ | $0 \cdot 855$ | 0.847 | 0.840 | 0.833 | 1 |
| 2 | 1.713 | 1.690 | 1.668 | 1.647 | 1.626 | 1.605 | 1.585 | 1.566 | 1.547 | 1.528 | 2 |
| 3 | $2 \cdot 444$ | $2 \cdot 402$ | $2 \cdot 361$ | $2 \cdot 322$ | $2 \cdot 283$ | $2 \cdot 246$ | $2 \cdot 210$ | $2 \cdot 174$ | $2 \cdot 140$ | $2 \cdot 106$ | 3 |
| 4 | $3 \cdot 102$ | 3.037 | 2.974 | 2.914 | $2 \cdot 855$ | $2 \cdot 798$ | $2 \cdot 743$ | $2 \cdot 690$ | 2.639 | 2.589 | 4 |
| 5 | 3.696 | 3.605 | 3.517 | 3.433 | 3.352 | $3 \cdot 274$ | $3 \cdot 199$ | $3 \cdot 127$ | 3.058 | 2.991 | 5 |
| 6 | 4.231 | 4.111 | 3.998 | 3.889 | $3 \cdot 784$ | 3.685 | 3.589 | 3.498 | 3.410 | $3 \cdot 326$ | 6 |
| 7 | $4 \cdot 712$ | 4.564 | $4 \cdot 423$ | $4 \cdot 288$ | 4.160 | 4.039 | 3.922 | 3.812 | 3.706 | 3.605 | 7 |
| 8 | $5 \cdot 146$ | 4.968 | $4 \cdot 799$ | 4.639 | 4.487 | 4.344 | $4 \cdot 207$ | 4.078 | 3.954 | 3.837 | 8 |
| 9 | 5.537 | $5 \cdot 328$ | $5 \cdot 132$ | 4.946 | 4.772 | $4 \cdot 607$ | $4 \cdot 451$ | 4.303 | $4 \cdot 163$ | 4.031 | 9 |
| 10 | 5.889 | $5 \cdot 650$ | $5 \cdot 426$ | $5 \cdot 216$ | 5.019 | $4 \cdot 833$ | $4 \cdot 659$ | 4.494 | $4 \cdot 339$ | 4.192 | 10 |
| 11 | $6 \cdot 207$ | 5.938 | $5 \cdot 687$ | $5 \cdot 453$ | 5.234 | 5.029 | $4 \cdot 836$ | $4 \cdot 656$ | $4 \cdot 486$ | 4.327 | 11 |
| 12 | $6 \cdot 492$ | $6 \cdot 194$ | 5.918 | 5.660 | $5 \cdot 421$ | $5 \cdot 197$ | 4.988 | $4 \cdot 793$ | 4.611 | 4.439 | 12 |
| 13 | 6.750 | 6.424 | $6 \cdot 122$ | $5 \cdot 842$ | 5.583 | $5 \cdot 342$ | $5 \cdot 118$ | 4.910 | $4 \cdot 715$ | 4.533 | 13 |
| 14 | 6.982 | 6.628 | $6 \cdot 302$ | 6.002 | $5 \cdot 724$ | $5 \cdot 468$ | $5 \cdot 229$ | 5.008 | 4.802 | 4.611 | 14 |
| 15 | $7 \cdot 191$ | $6 \cdot 811$ | $6 \cdot 462$ | $6 \cdot 142$ | 5.847 | 5.575 | $5 \cdot 324$ | 5.092 | $4 \cdot 876$ | $4 \cdot 675$ | 15 |

Standard normal distribution table

|  | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | $0 \cdot 09$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| $0 \cdot 1$ | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| $0 \cdot 2$ | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | $0 \cdot 1026$ | $0 \cdot 1064$ | $0 \cdot 1103$ | $0 \cdot 1141$ |
| $0 \cdot 3$ | $0 \cdot 1179$ | $0 \cdot 1217$ | $0 \cdot 1255$ | $0 \cdot 1293$ | $0 \cdot 1331$ | $0 \cdot 1368$ | $0 \cdot 1406$ | 0.1443 | 0.1480 | $0 \cdot 1517$ |
| 0.4 | $0 \cdot 1554$ | $0 \cdot 1591$ | $0 \cdot 1628$ | $0 \cdot 1664$ | $0 \cdot 1700$ | $0 \cdot 1736$ | $0 \cdot 1772$ | $0 \cdot 1808$ | $0 \cdot 1844$ | $0 \cdot 1879$ |
| 0.5 | $0 \cdot 1915$ | 0.1950 | $0 \cdot 1985$ | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | $0 \cdot 3051$ | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | $0 \cdot 3159$ | 0.3186 | $0 \cdot 3212$ | $0 \cdot 3238$ | $0 \cdot 3264$ | $0 \cdot 3289$ | 0.3315 | 0.3340 | $0 \cdot 3365$ | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | $0 \cdot 3508$ | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| $1 \cdot 1$ | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| $1 \cdot 2$ | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | $0 \cdot 3997$ | 0.4015 |
| $1 \cdot 3$ | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | $0 \cdot 4222$ | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| $2 \cdot 1$ | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| $2 \cdot 2$ | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| $2 \cdot 3$ | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| $2 \cdot 5$ | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| $2 \cdot 6$ | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| $2 \cdot 7$ | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| $2 \cdot 9$ | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

This table can be used to calculate $N(d)$, the cumulative normal distribution functions needed for the Black-Scholes model of option pricing. If $d_{i}>0$, add 0.5 to the relevant number above. If $d_{i}<0$, subtract the relevant number above from 0.5 .

## End of Question Paper

